GAS FLOW PAST NONAXISYMMETRIC BODIES AT SMALL ANGLES OF ATTACK

PMM Vol. 34, №1, 1970, pp. 137-140)
V. N. MIKHAILOV
(Moscow)
(Received August 4, 1969)

The theory of small perturbations is used extensively in studying flow past bodies at small angles of attack α. The solution in this case is a sum of two terms, one of which represents the solution for $\alpha = 0$, while the other is proportional to α. If linearization is effected near the unperturbed flow, the second terms take the form of linear equations with constant coefficients. This theory is valid for slender bodies and wings. In the case of thick-section bodies or surfaces, linearization can be carried out near the known perturbed flow only, e.g. near the axisymmetric flow [1-3]. In this case the coefficients of the linear equations for the second terms depend on the first terms of the series expansion in α.

If the body is nonaxisymmetric, the second terms depend on the orientation of the body relative to the free-flow velocity vector. We show that the solution can be found for any position of the body in this case, provided one knows the solution for just two positions of the body relative to the free flow. A similar result for specific slender bodies was arrived at in [4, 5].

1. Let us consider the flow past some finite body of a stream of ideal gas with the constant velocity V_∞, pressure p_∞ and density ρ_∞. We shall use the rectangular coordinate system x, y, z rigidly attached to the body and the system x, y', z' such that the free-flow velocity vector lies in the plane of the axes x, y' (see Fig. 1). These two systems are related by the equations

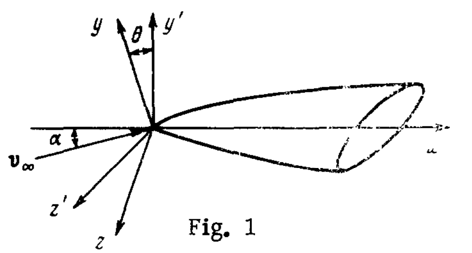

Fig. 1

$$y' = y\cos\theta + z\sin\theta$$
$$z' = -y\sin\theta + z\cos\theta \qquad (1.1)$$

Here θ is the angle between the axes y and y' (the roll angle).

The angle between the free-flow velocity vector and the x-axis is α (the angle of attack).

The projections u, v, w of the velocity vector on the x-, y- and z-axes, respectively, the pressure p, and the density ρ must satisfy the equations

$$^1/_2 \operatorname{grad}(\mathbf{V}^2) - \mathbf{V}\times\operatorname{rot}\mathbf{V} = -\rho^{-1}\operatorname{grad} p$$
$$\operatorname{div}(\rho\mathbf{V}) = 0, \qquad \mathbf{V}\operatorname{grad}(p/\rho^\varkappa) = 0 \qquad (1.2)$$

At infinity we have

$$u = V_\infty\cos\alpha,\ v = V_\infty\sin\alpha\cos\theta,\ w = V_\infty\sin\alpha\sin\theta,\ p = p_\infty,\ \rho = \rho_\infty \quad (1.3)$$

The normal component of the velocity is equal to zero at the body. If $F(x, y, z) = 0$ is the equation of the body surface, this condition can be written as

$$\mathbf{V}\cdot\operatorname{grad} F = 0 \qquad (1.4)$$

2. We shall attempt to find the solution of Eqs. (1.2) for small angles α in the form

$$u = u_0 + \alpha u', \; v = v_0 + \alpha v', \; w = w_0 + \alpha w' \tag{2.1}$$

$$p = p_0 + \alpha p', \; \rho = \rho_0 + \alpha \rho'$$

Here the functions $u_0, v_0, w_0, p_0, \rho_0$ are the solution of the problem for $\alpha = 0$. They satisfy Eqs. (1.2) and boundary conditions (1.3) and (1.4) in which we have set $\alpha = 0$.

Substituting relation (2.1) into Eqs. (1.2), we obtain the following equations for the quantities u', v', w', p', ρ':

$$\operatorname{grad}(\mathbf{V}_0\mathbf{V}') - \mathbf{V}_0 \times \operatorname{rot}\mathbf{V}' - \mathbf{V}' \times \operatorname{rot}\mathbf{V}_0 = \frac{\rho'}{\rho_0^2}\operatorname{grad} p_0 - \frac{1}{\rho_0}\operatorname{grad} p'$$

$$\operatorname{div}(\rho_0\mathbf{V}' + \rho'\mathbf{V}_0) = 0, \qquad \mathbf{V}'\cdot\operatorname{grad}\frac{p_0}{\rho_0^{\varkappa}} + \mathbf{V}_0\cdot\operatorname{grad}\left[\frac{p_0}{\rho_0^{\varkappa}}\left(\frac{p'}{p_0} - \varkappa\frac{\rho'}{\rho_0}\right)\right] = 0 \tag{2.2}$$

Linearization of boundary conditions (1.3) and (1.4) yields the following conditions for the primed parameters:

$$u' = 0, \; v' = \alpha V_\infty \cos\theta, \; w' = \alpha\, V_\infty \sin\theta, \;\; p' = 0, \; \rho' = 0 \tag{2.3}$$

$$u'\frac{\partial F}{\partial x} + v'\frac{\partial F}{\partial y} + w'\frac{\partial F}{\partial z} = 0 \tag{2.4}$$

We note that in the coordinate system x, y, z the null terms in expressions (2.1) are the same for differing θ, since boundary conditions (1.3) do not depend on θ for $\alpha = 0$ The equation of the body surface is also independent of θ in these coordinates.

Let us denote the solutions of Eqs. (2.2) for $\theta = 0$ by the subscript 1 and for $\theta = {}^1/_2\,\pi$ by the subscript 2. These solutions satisfy the conditions

$$u_1' = 0, \quad v_1' = V_\infty, \; w_1' = 0, \; p_1' = 0, \; \rho_1' = 0 \tag{2.5}$$

$$u_2' = 0, \; v_2' = 0, \; w_2' = V_\infty, \; p_2' = 0, \; \rho_2' = 0$$

at infinity and condition (2.4) at the body.

Equations (2.2) are linear in the parameters u', v', w', p', ρ', so that the functions

$$u' = u_1'\cos\theta + u_2'\sin\theta, \; v' = v_1'\cos\theta + v_2'\sin\theta, \;\; w' = w_1'\cos\theta + w_2'\sin\theta$$

$$p' = p_1'\cos\theta + p_2'\sin\theta, \qquad \rho' = \rho_1'\cos\theta + \rho_2'\sin\theta \tag{2.6}$$

are the solutions of these equations. They also satisfy condition (2.4) at the body and boundary conditions (1.3) at infinity by virtue of Eqs. (2.5). We have therefore proved the following theorem: if the problem has a solution for $\theta = 0$ and $\theta = {}^1/_2\,\pi$, then there also exists a solution for any angle θ, and this solution can be expressed in terms of the two aforementioned solutions by way of formulas (2.6).

3. Conditions at the resulting shock waves must be fulfilled in supersonic flow past a body. We can show that the solution can be expressed in the form (2.6) in this case as well.

The conditions at the shock wave in three-dimensional flow are

$$-[\rho V_n\,\mathbf{V}] = [p]\,\mathbf{n}, \qquad [\rho V_n] = 0 \tag{3.1}$$

$$-\left[\rho V_n\left(\frac{1}{2}V^2 + \frac{1}{\varkappa - 1}\frac{p}{\rho}\right)\right] = [pV_n]$$

The square brackets indicate that the quantities inside them experience jumps in passing through the shock wave; V_n is the velocity component along the normal to the shock wave surface.

Let $G(x, y, z) = 0$ be the equation of the shock wave surface; the unit vector of the normal n to this surface can then be written as

$$\mathbf{n} = \frac{\operatorname{grad} G}{\sqrt{(\operatorname{grad} G)^2}}$$

so that $V_n = \mathbf{V}\cdot\mathbf{n}$. This enables us to rewrite expressions (3.1) as

$$[\rho\mathbf{V}] \operatorname{grad} G = 0, \quad -[\rho\,(\mathbf{V}\cdot\operatorname{grad} G)\,\mathbf{V}] = [p] \operatorname{grad} G \tag{3.2}$$

$$-\left[\rho\,(\mathbf{V}\cdot\operatorname{grad} G)\left(\frac{1}{2}V^2 + \frac{1}{\varkappa-1}\frac{p}{\rho}\right)\right] = [p\mathbf{V}] \operatorname{grad} G$$

The equation of the shock wave surface for small α can be written as

$$G(x, y, z) = G_0(x, y, z) + \alpha' G'(x, y, z) \tag{3.3}$$

Here $G_0(x, y, z) = 0$ is the equation for the shock wave surface for $\alpha = 0$, which is independent of the angle θ in the coordinate system x, y, z.

Substituting relations (3.3) and (2.1) into expressions (3.2), we obtain the following equations for the linear terms:

$$[\rho_0\mathbf{V}_0 \operatorname{grad} G' + (\rho_0\mathbf{V}' - \rho'\mathbf{V}_0) \operatorname{grad} G_0] = 0$$

$$[-(\rho' V_0 \operatorname{grad} G_0 + \rho_0\mathbf{V}' \operatorname{grad} G_0 + \rho_0\mathbf{V}_0 \operatorname{grad} G')\,\mathbf{V}_0] -$$
$$-[\rho_0\,(\mathbf{V}_0 \operatorname{grad} G_0)\,\mathbf{V}'] = [p'] \operatorname{grad} G_0 + [p_0] \operatorname{grad} G'$$

$$-\left[(\rho'\mathbf{V}_0 - \rho_0\mathbf{V}') \operatorname{grad}\left(\frac{1}{2}V_0^2 + \frac{1}{\varkappa-1}\frac{p_0}{\rho_0}\right)\right] - \tag{3.4}$$
$$-\left[\rho_0\mathbf{V}_0\cdot\operatorname{grad}\left(\mathbf{V}_0\mathbf{V}' + \frac{1}{\varkappa-1}\frac{p_0}{\rho_0}\left(\frac{p'}{p_0} - \frac{\rho'}{\rho_0}\right)\right)\right] =$$
$$= [p_0\mathbf{V}_0] \operatorname{grad} G' + [p_0\mathbf{V}' + p'\mathbf{V}_0] \operatorname{grad} G_0$$

Let us substitute solution (2.6) into the first equation of system (3.4),

$$[\rho_0\mathbf{V}_0 \operatorname{grad} G' + (\rho_0\mathbf{V}_1' - \rho'\mathbf{V}_0) \operatorname{grad} G_0 \cos\theta + \sin\theta\,(\rho_0\mathbf{V}_2' - \rho_2'\mathbf{V}_0)\times$$
$$\times \operatorname{grad} G_0] = 0 \tag{3.5}$$

Taking the function G' in the form

$$G' = G_1' \cos\theta + G_2' \sin\theta \tag{3.6}$$

where G_1' and G_2' are functions which yield the solutions for $\theta = 0$ and $\theta = {}^1/_2\,\pi$, we ensure identical fulfillment of Eq. (3.5), since the quantities with the subscripts 1 and 2 satisfy relations (3.4). In exactly the same way we can show that the remaining equations of (3.4) are satisfied, provided the solution for any angle θ is taken in the form (2.6) and the shock wave surface is given by (3.3), (3.6).

4. Let us denote the forces acting along the y- and z-axes by Y and Z, respectively. By virtue of (2.6) these forces are given by

$$Y = Y_0 + \alpha\,(Y_1' \cos\theta + Y_2' \sin\theta), \qquad Z = Z_0 + \alpha\,(Z_1' \cos\theta + Z_2' \sin\theta) \tag{4.1}$$

The subscripts in these expressions have the same significance as above.

The aerodynamic force components N and R acting along the y'- and z'-axes can be expressed in terms of Y and Z,

$$N = Y\cos\theta + Z\sin\theta, \qquad R = Z\cos\theta - Y\sin\theta \tag{4.2}$$

Substituting (4.1) into (4.2), we obtain

$$N = Y_0 \cos\theta + Z_0 \sin\theta + \alpha\,[N_1' \cos^2\theta + N_2' \sin^2\theta + (R_1' - R_2') \sin\theta \cos\theta] \tag{4.3}$$

$$R = Z_0 \cos\theta - Y_0 \sin\theta + \alpha\,[R_1' \cos^2\theta + R_2' \sin^2\theta + (N'_2 - N_1') \sin\theta \cos\theta]$$

Here

$$N_1' = Y_1', \qquad N_2' = Z_2', \qquad R_1' = Z_1', \qquad R_2' = -Y_2'$$

Formulas (4.3) express the normal and lateral forces for any roll angle in terms of their values for $\theta = 0$ and $\theta = \frac{1}{2}\pi$.

Let us consider bodies symmetric upon rotation by the angle $\Delta\theta = 2\pi/n$ (n is an integer). Rotation of the body of the angle $\theta_i = i2\pi/n$ then ensures fulfillment of the equations $N(\theta_i) = N(0)$, $R(\theta_i) = R(0)$. Hence, relations (4.3) yield the equations

$$(N_2' - N_1') \sin^2\theta_i + (R_2' - R_1') \sin\theta_i \cos\theta_i = 0$$

$$(N_2' - N_1') \cos\theta_i \sin\theta_i + (R_2' - R_1') \sin^2\theta_i = 0$$

For $n > 2$ these equations have a zero solution only and formulas (4.3) become

$$N = \alpha N_1', \qquad R = \alpha R_1' \tag{4.4}$$

Hence, the normal and lateral forces do not depend on the roll angle in this case.

If the body is also specularly symmetric, e.g. for $\theta = 0$, then $R_1' = 0$ and we infer from (4.4) that $R = 0$ for any roll angle.

The above results can be extended to the case of harmonic vibrations of bodies.

BIBLIOGRAPHY

1. Stone, A., Supersonic motion of a cone at a small angle of attack, (Russian translation). In collection: Gas Dynamics. Moscow, Izd. inostr. lit., 1950.
2. Kolodochkin, V. P., Analysis of supersonic flow past sharpnosed solids of revolution in relation to the angle of attack. Izv. Akad. Nauk SSSR, OTN №1, 1962.
3. Telenin, G. F. and Lipnitskii, Iu. M., Unsteady supersonic flow past blunt-nosed bodies with a detached shock wave. Izv. Akad. Nauk SSSR, MZhG №4, 1966.
4. Spreiter, I. R., The aerodynamic forces on slender plane and cruciform wing and body combinations. NACA Rep., №962, 1950.
5. Shurygin, V. M., V-shaped wings in supersonic flow. In: Collected Papers on Aerodynamics. Moscow, Oborongiz, 1957.

Translated by A. Y.